# Topos Theory - Course Summary

Here is a summary of some of the main topics we discussed with links to where you can find them in the two main sources I used: [2] which I will write as MM and [1] which I will write as JTT.

1. Motivation
   (a) Foundational - Find axioms characterizing the category of sets. Can they usefully be generalized?
   (b) Model Theoretic - Find counter models for proving independence resulsts ("forcing")
   (c) Exotic Sets - Categories where, say, the sets of real number $\mathbb{R}$ have different properties than they do classically.
   (d) Semantics of Programming Languages - Alternatives to $\mathcal{S}et$ which better model computational behavior. Can be used to prove normalization results.
   (e) Topology and Algebra - Natural home for many interesting invariants in algebra and geometry.
2. Definitions of Elementary Topoi
   - MM, Chapter 4.
   - JTT, Chapter 1
3. Monos are Equalizers. Topoi are Balanced.
   - JTT, Seciton 1.2
   - MM, Proposition 2, p. 167
4. Construction of Finite Colimits
   - MM, Section 4.5, p. 180
   - JTT, Theorem 1.34 and Corollay 1.36, p. 33-34
5. Slice category is a Topos
   - MM, Section 4.7
   - JTT, Theorem 1.42, p. 35

6. Image Factorization

    - MM, Section 4.6
    - JTT, Section 1.5, p. 40

7. Heyting Algebra Structure of Subobjects

    - MM, Section 4.6, Proposition 3, p. 186
      and Section 4.8
    - JTT, Proposition 5.13, p. 137

8. Open Objects of a Topos

    - MM, Section 4.6, p. 189.

# References


[1] Peter T Johnstone. *Topos theory*. Courier Corporation, 2014.

[2] Saunders MacLane and Ieke Moerdijk. *Sheaves in geometry and logic: A first introduction to topos theory*. Springer Science & Business Media, 2012.