

## Homotopy Type Theory - Exercise Sheet 4

1. Show that

$$S^1 \simeq \text{Susp } \mathbb{B}\text{ool}$$

where  $S^1$  is the circle defined using `base` and `loop`.

2. Define the *fundamental group* of a pointed type  $(X, x)$  as

$$\pi_1(X, x) := |x \equiv x|_0$$

Show that  $\pi_1(X, x)$  is a group.

3. Show that  $X$  is a proposition if and only if  $X \rightarrow X$  is contractible.
4. Let  $(X, x)$  be a pointed type. We write  $\Omega(X, x)$  for the *loop space* of  $(X, x)$  which is the pointed type  $(x \equiv x, \text{refl})$ . Write  $\Omega^n(X, x)$  for the  $n$ -th iteration of this construction (and write  $\Omega^0(X, x) := (X, x)$ ).

Show that a type  $X$  is an  $n$ -type (for  $n \geq -1$ ) if and only if  $\Omega^{n+1}(X, x)$  is contractible for all  $x : X$ . *Hint:* work by induction.

5. Show that the  $n$ -th truncation  $| - |_n$  preserves products:

$$|X \times Y|_n \simeq |X|_n \times |Y|_n$$