Topos Theory - Exercise Sheet 3

1. Use the fact that if $\mathcal{E}$ is a topos then $\mathcal{E}/X$ is also a topos for any object $X \in \mathcal{E}$ to show that the set $\mathrm{Sub}(X)$ of subobjects of X can be endowed with the structure of a Heyting algebra.
2. Check that the formula
$$G^F(C) := \mathrm{Hom}_{\mathcal{S}et^{\mathcal{C}^{op}}}(yC \times F, G)$$
makes G^F into an exponential in $\mathcal{S}et^{\mathcal{C}^{op}}$.
3. Check that the formula
$$\Omega(C) := \{S \mid S \text{ is a sieve in } \mathcal{C}\}$$
makes Ω into a subobject classifier.